CLASSES BY SACHIN SHARMA

DIFFERENTIATION - Exercise 5.2

Derivative of Trigonometric functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\text{cosec}^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\text{cosec}\, x = -\text{cosec}\, x \cot x$$

Note:
Derivative of cosx, cosecx and cotx is **NEGATIVE.**

Derivative of INVERSE Trigonometric functions

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\mathrm{cosec}^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$$

Note: Derivative of **$\cos^{-1}x$, $\mathrm{cosec}^{-1}x$ and $\cot^{-1}x$** is **NEGATIVE**.

Derivative of Algebraic Function

$$\frac{d}{dx} x^n = nx^{n-1}$$

Examples:

$$\frac{d}{dx} x^3 = 3x^{3-1} = 3x^2$$

$$\frac{d}{dx} x^{-3} = -3x^{-3-1} = -3x^{-4} = \frac{-3}{x^4}$$

$$\frac{d}{dx} x = x^1 = 1 \times x^{1-1} = x^0 = 1$$

$$\frac{d}{dx} x^{3/2} = \frac{3}{2} x^{3/2-1} = \frac{3}{2} x^{1/2}$$

$$\frac{d}{dx} \sqrt{x} = \frac{d}{dx} x^{½} = \frac{1}{2} x^{1/2-1} = \frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

Derivative of **Exponent** functions

$$\frac{d}{dx}e^{x}= e^{x}$$

$$\frac{d}{dx}e^{2x}= e^{2x} \times 2 = 2e^{2x}$$

$$\frac{d}{dx}e^{-2x}=e^{-2x} \times(-2)=-2e^{-2x}$$

Derivative of **Log** functions

$$\frac{d}{dx}\text{Log } x=\frac{1}{x}$$

$$\frac{d}{dx}\text{Log } 2x=\frac{1}{2x}\times 2 = \frac{1}{x}$$

Some other derivative

$$\frac{d}{dx}a^{x}= a^{x}\log a$$

$$\frac{d}{dx}5^{x}= 5 \times \log 5$$

Derivative of **constant**

$$\frac{d}{dx}5 = 0$$

$$\frac{d}{dx}a = 0$$

CLASS 12TH - Ch-5

Exercise 5.2

Basic Exercise for understanding differentiation

Q1: $\sin(x^2+5)$

I II

Let $y = \sin(x^2+5)$

$$\frac{dy}{dx} = \cos(x^2+5) \times 2x$$

$$= 2x \cos(x^2+5)$$

- Here we have function (function)
- First function is sin (x^2+5) and second function is x^2+5
- So, derivative of sin (x^2+5) is cos (x^2+5)
- and derivative of x^2+5 is $2x + 0 = 2x$

Q2: Cos(Sin x)

I II III

Let y = Cos (Sin x)

$\frac{dy}{dx}$ = -Sin(**Sin x)** × **Cos x**

= -Sin(**Sin x)** **Cos x**

यहाँ function के अंदर function के अंदर function है मतलब तीन function है

पहला function cos, दूसरा function sin और तीसरा **x** है

जैसे cos x का derivative –sin x होता है वैसे ही cos (sin x) का derivative –sin (sin x) **होगा**

Now अंदर वाले sin x का derivative होगा cos x

and lastly x का derivative 1 होगा | लेकिन अगर ये होता **Cos(Sin x^2) then we will write 2x**

Q4: $Sec(\tan\sqrt{x})$

I II III IV

Let $y = Sec(\tan\sqrt{x})$

I II III IV

$$\frac{dy}{dx} = Sec(\tan\sqrt{x})\,Tan(\tan\sqrt{x}) \times Sec^2\sqrt{x} \times \frac{1}{2\sqrt{x}} \times 1 = \frac{1}{2\sqrt{x}}\,Sec(\tan\sqrt{x})\,Tan(\tan\sqrt{x})Sec^2\sqrt{x}$$

Here we have total 4 functions.

First function sec, **Second** function tan, **Third** $\sqrt{\ }$ and **Fourth** is **x**

Derivative of sec x is sec **x** tan **x**

so derivative of Sec**($\tan\sqrt{x}$)** will be sec**($\tan\sqrt{x}$)** tan**($\tan\sqrt{x}$)**

Now the second function is $\tan\sqrt{x}$

Derivative of tan**(x)** is sec**(x)** tan**(x)**, **So** Derivative of tan**(**$\sqrt{x}$**)** will be sec**(**$\sqrt{x}$**)** tan**(**$\sqrt{x}$)

Third is $\sqrt{x}$ and Fourth is x

Q5: $\dfrac{\sin(ax+b)}{\cos(cx+d)}$

Let $y = \dfrac{\sin(ax+b)}{\cos(cx+d)}$

$$\frac{dy}{dx} = \frac{\cos(cx+d)\,\frac{d}{dx}\sin(ax+b) - \sin(ax+b)\,\frac{d}{dx}\cos(cx+d)}{\cos^2(cx+d)}$$

$$\frac{dy}{dx} = \frac{\cos(cx+d\ \cos(ax+b)\times a - \sin(ax+b)\ (-\sin(cx+d)) * c}{\cos^2(cx+d)}$$

$$= a\ \sec(cx+d)\cos(ax+b) - c\ \sin(ax+b)\ \tan(cx+d)\ \sec\ (cx+d)$$

QUOTIENT RULE: $\dfrac{Q\ (\text{derivative of } P) - P\ (\text{derivative of } Q)}{Q^2}$

Q6: $\text{Cos}\, x^3\ \text{Sin}^2\, x^5$

I II

Let y $= \text{Cos}\, x^3\ \text{Sin}^2\, x^5$

$$\frac{dy}{dx} = \text{Cos}\, x^3\ (2\ \text{Sin}x^5 \times \text{Cos}x^5\ 5\, x^4) \quad + \quad \text{Sin}^2 x^5\ (-\text{Sin}x^3 * 3x^2)$$

$$= 10\, x^4\ \text{Cos}\, x^3\ \text{Sin}\, x^5\ \text{Cos}\, x^5 \quad - \quad \text{Sin}x^3 \times 3x^2\ (\text{Sin}^2 x^5)$$

PRODUCT RULE: I (Derivative of 2nd) + II (Derivative of 1st)

यहाँ दो function -product की form में given है (like A*B) so we will apply Product rule of derivative.

PRODUCT RULE:

1st function as it is * (derivative of 2nd function) + 2nd function as it is * (derivative of 1st)

As there is a + sign in between so we can also write as :-

2nd function as it is * (derivative of 1st) + 1st function as it is * (derivative of 2nd function)

Answer will be the same in both the expression.

Q7: $2\sqrt{\cot(x^2)}$

I II III

Let $y = 2\sqrt{\cot(x^2)}$

$$\frac{dy}{dx} = 2 \times \frac{1}{2\sqrt{\cot(x^2)}} \cdot -\text{Cosec}(x^2) \times 2x = \frac{-2 \times \text{Cosec}(x^2)}{2\sqrt{\cot(x^2)}}$$

यहाँ सबसे पहले $\sqrt{}$ का derivative **होगा**। जैसे $\sqrt{x}$ का derivative $\frac{1}{2\sqrt{x}}$ होता है

वैसे ही $\sqrt{\cot(x^2)}$ का derivative $\frac{1}{2\sqrt{\cot(x^2)}}$ होगा

फिर $\cot x^2$ का derivative and lastly x^2 का derivative होगा